

**Nonperturbative corrections to moments of the decay  $B \rightarrow X_s \ell^+ \ell^-$** 

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**Abstract**

We study nonperturbative corrections to the inclusive rare decay  $B \rightarrow X_s \ell^+ \ell^-$  by performing an operator product expansion (OPE) to  $\mathcal{O}(1/m_b^3)$ . The values of the matrix elements entering at this order are unknown and introduce uncertainties into physical quantities. We study uncertainties introduced into the partially integrated rate, moments of the hadronic spectrum, as well as the forward-backward asymmetry. We find that for large dilepton invariant mass  $q^2 > M_{\psi'}^2$ , these uncertainties are large. We also assess the possibility of extracting the HQET parameters  $\lambda_1$  and  $\bar{\Lambda}$  using data from this process.

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## I. INTRODUCTION

Rare  $B$  decays mediated via flavour changing neutral currents have received much attention because of their sensitivity to physics beyond the standard model. In the standard model these decays occur via penguin and box diagrams with virtual electroweak bosons and up-type quarks in the loops. Because of the large top quark mass, the contribution with a top quark in the loop dominates. At energy scales below the mass of the top quark and  $W$  boson, it is convenient to switch to an effective theory where the top quark and the weak bosons have been integrated out of the theory. The  $b \rightarrow s$  transition is then mediated by the effective Hamiltonian [1]

$$\mathcal{H}_{eff} = -\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu)\mathcal{O}_i(\mu), \quad (1)$$

where the operators are commonly defined by

$$\begin{aligned} \mathcal{O}_1 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha})(\bar{c}_{L\beta}\gamma^\mu c_{L\beta}), \\ \mathcal{O}_2 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta})(\bar{c}_{L\beta}\gamma^\mu c_{L\alpha}), \\ \mathcal{O}_3 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta}\gamma^\mu q_{L\beta}), \\ \mathcal{O}_4 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{L\beta}\gamma^\mu q_{L\alpha}), \\ \mathcal{O}_5 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\alpha}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta}\gamma^\mu q_{R\beta}), \\ \mathcal{O}_6 &= (\bar{s}_{L\alpha}\gamma_\mu b_{L\beta}) \sum_{q=u,d,s,c,b} (\bar{q}_{R\beta}\gamma^\mu q_{R\alpha}), \\ \mathcal{O}_7 &= \frac{e}{16\pi^2} \bar{s}_\alpha \sigma_{\mu\nu} (m_b R + m_s L) b_\alpha F^{\mu\nu}, \\ \mathcal{O}_8 &= \frac{g}{16\pi^2} \bar{s}_\alpha T_{\alpha\beta}^a \sigma_{\mu\nu} (m_b R + m_s L) b_\beta G^{a\mu\nu}, \\ \mathcal{O}_9 &= \frac{e^2}{16\pi^2} \bar{s}_\alpha \gamma^\mu L b_\alpha \bar{\ell} \gamma_\mu \ell, \\ \mathcal{O}_{10} &= \frac{e^2}{16\pi^2} \bar{s}_\alpha \gamma^\mu L b_\alpha \bar{\ell} \gamma_\mu \gamma_5 \ell. \end{aligned} \quad (2)$$

Here  $L/R = \frac{1}{2}(1 \pm \gamma_5)$  are the usual left and right handed chiral projection operators. The values of the Wilson coefficients  $C_i(m_b)$  have been calculated in the next to leading log approximation [2,3] in the standard model and are given in Table I.

$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7$	$C_9$	$C_{10}$
-0.240	1.103	0.011	-0.025	0.007	-0.030	-0.311	4.153	-4.546

TABLE I. The Wilson coefficients  $C_i(m_b)$  in the next-to-leading log approximation.

Physics beyond the standard model will generally introduce new contributions to the loop and will therefore modify the values of these coefficients [4]. Measurements of the Wilson coefficients may therefore indirectly constrain new physics scenarios. For example,

the decay  $b \rightarrow s\gamma$  is proportional to  $|C_7|^2$ , and the recent measurements of the branching ratio  $B \rightarrow K^*\gamma$  [5] and inclusive rate  $B \rightarrow X_s\gamma$  [6] have placed constraints on models of physics beyond the standard model which modify the magnitude of  $C_7$  [7].

The decay  $b \rightarrow s\ell^+\ell^-$  is suppressed, relative to  $b \rightarrow s\gamma$ , by an additional factor of the electromagnetic coupling constant and has not yet been observed [8]. It has, however, the appeal of being sensitive to the signs and magnitudes of  $C_7$ ,  $C_9$ , and  $C_{10}$ , making it a potentially more powerful probe than  $b \rightarrow s\gamma$  of beyond the standard model physics. Experimental studies of this process impose cuts on the available phase space. This is primarily due to the necessity of removing the resonance from  $B \rightarrow (J/\psi, \psi')X_s$  with the  $(J/\psi, \psi')$  decaying into two leptons. We incorporate representative cuts into the theoretical analysis.

Using an operator product expansion (OPE) several observables of the inclusive decay  $B \rightarrow X_s\ell^+\ell^-$  have been calculated including the leading non-perturbative corrections [9,10]. In this framework these leading corrections arise as matrix elements of dimension five operators, suppressed by two powers of the  $b$  quark mass, and are conventionally parameterized by two quantities,  $\lambda_1$  and  $\lambda_2$ . A third parameter  $\bar{\Lambda}$  enters through the difference of the  $b$  quark mass and  $B$  meson mass

$$m_b = M_B - \bar{\Lambda} + \frac{\lambda_1 + 3\lambda_2}{2m_b} + \dots \quad (3)$$

Whereas  $\lambda_2$  can be determined from the  $B^* - B$  mass splitting,  $\lambda_2 = (M_{B^*}^2 - M_B^2)/4 \simeq 0.12 \text{ GeV}^2$ , no such simple relation exists for  $\lambda_1$ . It has been estimated using various methods to lie in the range  $0.1 \text{ GeV}^2 \leq (-\lambda_1) \leq 0.6 \text{ GeV}^2$  [12].

In a previous paper we extended the analysis of the total rate for  $B \rightarrow X_s\ell^+\ell^-$  to one order higher in the OPE [13]. The dimension six operators arising at this order can be parametrized by six quantities, commonly labelled  $\rho_{1-2}$  and  $\mathcal{T}_{1-4}$ , all of which are unknown. We found that the uncertainties introduced by these six parameters can be significant, depending primarily on the actual values of the matrix elements and the amount of accessible phase space. In this paper we give the details of that analysis and also present calculations for the forward-backward asymmetry and moments of the hadron invariant mass spectrum at  $\mathcal{O}(1/m_b^3)$ . As in our previous analysis, we neglect perturbative effects and effects due to the finite mass of the  $s$  quark, which have been considered elsewhere [10].

It has also been proposed that, rather than use this decay to search for new physics, it might instead be used to extract the parameters  $\bar{\Lambda}$  and  $\lambda_1$  through a measurement of its hadronic invariant mass moments [10]. We estimate the uncertainties in this extraction due to the unknown matrix elements of the dimension six operators.

This paper is organized as follows. In section II we briefly introduce the formalism used to calculate the nonperturbative corrections, and we present the results for the decay rate in section III. In section IV we calculate the forward-backward asymmetry of the lepton pair. We then proceed in section V to calculate moments of the hadronic invariant mass spectrum and estimate uncertainties in extracting  $\bar{\Lambda}$  and  $\lambda_1$  from these moments. Finally we discuss the results and state our conclusions.

## II. OPERATOR PRODUCT EXPANSION AND KINEMATICS

The procedure for calculating nonperturbative contributions to heavy hadron decays has been thoroughly discussed in the literature [14,15], and we present here only a brief outline of the technique. The differential rate is proportional to the product of a lepton tensor  $L_{\mu\nu}$  and a hadron tensor  $W^{\mu\nu}$  and for the process in question it may be written as

$$d\Gamma = \frac{1}{2M_B} \frac{G_F^2 \alpha^2}{2\pi^2} |V_{ts}^* V_{tb}|^2 d\Pi \left( L_{\mu\nu}^L W^{L\mu\nu} + L_{\mu\nu}^R W^{R\mu\nu} \right) \quad (4)$$

where  $\Pi$  denotes the three body phase space. The spin-summed tensor  $L_{\mu\nu}$  for massless leptons is

$$L_{L(R)}^{\mu\nu} = 2 \left[ p_+^\mu p_-^\nu + p_-^\mu p_+^\nu - g^{\mu\nu} p_+ \cdot p_- \mp i \epsilon^{\mu\nu\alpha\beta} p_{+\alpha} p_{-\beta} \right]. \quad (5)$$

The hadron tensor  $W^{\mu\nu}$  is related via the optical theorem to the imaginary part of the forward scattering matrix element  $W^{\mu\nu} = (-1/\pi) \text{Im} T^{\mu\nu}$  where

$$T_{\mu\nu}^{L(R)} = -i \int d^4x e^{-iq \cdot x} \langle B | T \{ J_\mu^{L(R)\dagger}(x), J_\nu^{L(R)}(0) \} | B \rangle. \quad (6)$$

In this equation  $J^\mu$  denotes the current mediating this transition, and is given by

$$J_{L(R)}^\mu = \bar{s} \left[ R \gamma^\mu \left( C_9^{eff} \mp C_{10} + 2C_7^{eff} \frac{\hat{q}}{\hat{q}^2} \right) + 2\hat{m}_s C_7^{eff} \gamma^\mu \frac{\hat{q}}{\hat{q}^2} L \right] b \quad (7)$$

where  $q \equiv (p_+ + p_-)$  is the dilepton momentum<sup>1</sup>. In accordance with convention, we have defined two effective Wilson coefficients:  $C_7^{eff} \equiv C_7 - C_5/3 - C_6$  and  $C_9^{eff}$ . The latter contains the operator mixing of  $O_{1-6}$  into  $O_9$  as well as the one loop matrix elements of  $O_{1-6,9}$  [2,3]. The full analytic expression for  $C_9^{eff}$  is quite lengthy and may be found in [3].

Since in the decay of a  $b$  quark the momentum transfer to the final state parton is large, the time-ordered product (6) can be expanded in terms of local operators [14,15]

$$-i \int d^4x e^{-iq \cdot x} T \{ J^\dagger(x), J(0) \} \sim \frac{1}{m_b} \left[ \mathcal{O}_0 + \frac{1}{2m_b} \mathcal{O}_1 + \frac{1}{4m_b^2} \mathcal{O}_2 + \frac{1}{8m_b^3} \mathcal{O}_3 + \dots \right], \quad (8)$$

where  $\mathcal{O}_n$  represents a set of local operators of dimension  $d = (3 + n)$ , each operator containing  $n$  derivatives. For a generic current  $J_\mu$  the expressions for these operators are quite lengthy. The complete set of operators for  $d \leq 5$  [16] and  $d = 6$  [17] appear in the literature. In this study we include operators up to and including dimension  $d = 6$ .

The standard Lorentz decomposition for the forward scattering amplitude is

$$T_{\mu\nu} = -T_1 g_{\mu\nu} + T_2 v_\mu v_\nu + T_3 i \epsilon_{\mu\nu\alpha\beta} v^\alpha \hat{q}^\beta + T_4 \hat{q}^\mu \hat{q}^\nu + T_5 (\hat{q}^\mu v^\nu + \hat{q}^\nu v^\mu), \quad (9)$$

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<sup>1</sup> Notice that this current  $J_\mu$  reduces to the  $(V - A)$  current when  $C_7^{eff} = 0$ ,  $C_9^{eff} = 1/2$ , and  $C_{10} = -1/2$ . This provides some useful cross-checks with known results for semileptonic  $B$  decays [18].

where  $v^\mu$  is the four-velocity of the initial  $b$  quark  $p_b = m_b v$ . Since in this paper we treat the final state leptons as massless ( $\ell = e, \mu$ ), the form factors  $T_{4-5}$  do not contribute to observables.

It is clear from (6) and (8) that to calculate these form factors we must take matrix elements of the operators  $\mathcal{O}_n$ . Matrix elements of dimension four operators vanish at leading order in the  $1/m_b$  expansion [14] and matrix elements of dimension five operators may be parameterized by  $\lambda_1$  and  $\lambda_2$  [19]

$$\langle B(v) | \bar{h}_v \Gamma i D_\mu i D_\nu h_v | B(v) \rangle = M_B \text{Tr} \left\{ \Gamma P_+ \left( \frac{1}{3} \lambda_1 (g_{\mu\nu} - v_\mu v_\nu) + \frac{1}{2} \lambda_2 i \sigma_{\mu\nu} \right) P_+ \right\}, \quad (10)$$

where  $P_+ = \frac{1}{2}(1 + \not{v})$ , and  $\Gamma$  is an arbitrary Dirac structure.

Finally, the dimension six operators may be parameterized by the matrix elements of two local operators [18,20]

$$\begin{aligned} \frac{1}{2M_B} \langle B(v) | \bar{h}_v i D_\alpha i D_\mu i D_\beta h_v | B(v) \rangle &= \frac{1}{3} \rho_1 (g_{\alpha\beta} - v_\alpha v_\beta) v_\mu, \\ \frac{1}{2M_B} \langle B(v) | \bar{h}_v i D_\alpha i D_\mu i D_\beta \gamma_\delta \gamma_5 h_v | B(v) \rangle &= \frac{1}{2} \rho_2 i \epsilon_{\nu\alpha\beta\delta} v^\nu v_\mu \end{aligned} \quad (11)$$

and by matrix elements of two time-ordered products

$$\begin{aligned} \frac{1}{2M_B} \langle B(v) | \bar{h}_v (iD)^2 h_v i \int d^3x \int_{-\infty}^0 dt \mathcal{L}_I(x) | B(v) \rangle + h.c. &= \frac{\mathcal{T}_1 + 3\mathcal{T}_2}{m_b}, \\ \frac{1}{2M_B} \langle B(v) | \bar{h}_v \frac{1}{2} (-i\sigma_{\mu\nu}) G^{\mu\nu} h_v i \int d^3x \int_{-\infty}^0 dt \mathcal{L}_I(x) | B(v) \rangle + h.c. &= \frac{\mathcal{T}_3 + 3\mathcal{T}_4}{m_b} \end{aligned} \quad (12)$$

arising from a mismatch between the states  $|B(v)\rangle$  of the effective theory and  $|B\rangle$  of the full theory. The contributions from  $\mathcal{T}_{1-4}$  can most easily be incorporated by making the replacements [18]

$$\begin{aligned} \lambda_1 &\rightarrow \lambda_1 + \frac{\mathcal{T}_1 + 3\mathcal{T}_2}{m_b} \\ \lambda_2 &\rightarrow \lambda_2 + \frac{\mathcal{T}_3 + 3\mathcal{T}_4}{3m_b} \end{aligned} \quad (13)$$

in the parton level results. In addition, as we will show later there is a contribution to the total rate from the dimension six four-fermion operator

$$O_{(V-A)}^{bs} = 16\pi^2 \left[ \bar{b} \gamma^\mu L s \bar{s} \gamma^\nu L b (g_{\mu\nu} - v_\mu v_\nu) \right], \quad (14)$$

the matrix element of which we define as

$$\frac{1}{2M_B} \langle B | O_{(V-A)}^{bs} | B \rangle \equiv f_1. \quad (15)$$

The form factors up to  $\mathcal{O}(1/m_b^2)$  have appeared in the literature [9]. The  $\mathcal{O}(1/m_b^3)$  contributions to the form factors proportional to  $\rho_{1-2}$  are presented in Appendix A. The dependence on  $\mathcal{T}_{1-4}$  is obtained by making the replacements (13) in the  $\mathcal{O}(1/m_b^2)$  form factors.

The triple differential branching ratio is given by

$$\begin{aligned}
\frac{d^3 \mathcal{B}}{d\hat{u} d\hat{s} dv \cdot \hat{q}} &= \frac{2 \mathcal{B}_0}{2M_B} \left( -\frac{1}{\pi} \right) \text{Im} \left\{ 2 \hat{s} \left( T_1^L(v \cdot \hat{q}, \hat{s}) + T_1^R(v \cdot \hat{q}, \hat{s}) \right) \right. \\
&\quad + \left( (v \cdot \hat{q})^2 - \hat{s} - \frac{\hat{u}^2}{4} \right) \left( T_2^L(v \cdot \hat{q}, \hat{s}) + T_2^R(v \cdot \hat{q}, \hat{s}) \right) \\
&\quad \left. + \hat{u} \hat{s} \left( T_3^L(v \cdot \hat{q}, \hat{s}) - T_3^R(v \cdot \hat{q}, \hat{s}) \right) \right\}, \tag{16}
\end{aligned}$$

where we have defined kinematic variables  $v \cdot \hat{q} = \frac{1}{m_b} v \cdot q$ ,  $\hat{s} = \frac{1}{m_b^2} q^2$ , and  $\hat{u} = \frac{1}{m_b^2} [(p_b - p_-)^2 - (p_b - p_+)^2]$ . In terms of these leptonic variables the limits of phase space are given by

$$\begin{aligned}
-\sqrt{\hat{s} + \frac{\hat{u}^2}{4}} &\leq v \cdot \hat{q} \leq \sqrt{\hat{s} + \frac{\hat{u}^2}{4}} \\
-\hat{u}(\hat{s}, \hat{m}_s) &\leq \hat{u} \leq \hat{u}(\hat{s}, \hat{m}_s) \\
4\hat{m}_l^2 &\leq \hat{s} \leq (1 - \hat{m}_s)^2, \tag{17}
\end{aligned}$$

where  $\hat{u}(\hat{s}, \hat{m}_s) = \sqrt{[\hat{s} - (1 + \hat{m}_s)^2][\hat{s} - (1 - \hat{m}_s)^2]}$ .

For the calculation of the hadron invariant mass moments it will be convenient to express the phase space in terms of the parton energy fraction  $x_0 = E_q/m_b$  and the parton invariant mass fraction  $\hat{s}_0 = p_q^2/m_b^2$ . They are related to the leptonic variables introduced above via

$$\begin{aligned}
v \cdot \hat{q} &= 1 - x_0 \\
\hat{s} &= 1 + \hat{s}_0 - 2x_0. \tag{18}
\end{aligned}$$

The phase space can then be expressed as

$$\begin{aligned}
-2\sqrt{x_0^2 - \hat{s}_0} &\leq \hat{u} \leq 2\sqrt{x_0^2 - \hat{s}_0} \\
\hat{m}_s^2 &\leq \hat{s}_0 \leq x_0^2 \\
\hat{m}_s &\leq x_0 \leq \frac{1}{2}(1 + \hat{m}_s^2). \tag{19}
\end{aligned}$$

Since the form factors  $T_i$  are independent of  $\hat{u}$ , this first integration is trivial and we arrive at

$$\begin{aligned}
\frac{d^2 \mathcal{B}}{dx_0 d\hat{s}_0} &= \frac{16 \mathcal{B}_0}{2M_B} \left( -\frac{1}{\pi} \right) \sqrt{x_0^2 - \hat{s}_0} \text{Im} \left\{ \left[ (1 - 2x_0 + \hat{s}_0) \left( T_1^L(x_0, \hat{s}_0) + T_1^R(x_0, \hat{s}_0) \right) \right. \right. \\
&\quad \left. \left. + \frac{x_0^2 - \hat{s}_0}{3} \left( T_2^L(x_0, \hat{s}_0) + T_2^R(x_0, \hat{s}_0) \right) \right] \right\} \tag{20}
\end{aligned}$$

In the above expressions we use the same conventions as in [9,10] and normalize the  $B \rightarrow X_s \ell^+ \ell^-$  branching ratio to the semileptonic branching ratio

$$d\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-) = \mathcal{B}_{sl} \frac{d\Gamma(B \rightarrow X_s \ell^+ \ell^-)}{\Gamma(B \rightarrow X_c \ell \nu_\ell)}, \tag{21}$$

introducing the normalization constant

$$\mathcal{B}_0 = \mathcal{B}_{sl} \frac{3\alpha^2}{16\pi^2} \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2} \frac{1}{f(\hat{m}_c) \kappa(\hat{m}_c)}. \quad (22)$$

In this expression  $f(\hat{m}_c)$  is the well-known phase space factor for the  $b \rightarrow ce\bar{\nu}$  parton decay rate

$$f(\hat{m}_c) = 1 - 8\hat{m}_c^2 + 8\hat{m}_c^6 - \hat{m}_c^8 - 24\hat{m}_c^4 \log \hat{m}_c \quad (23)$$

and  $\kappa(\hat{m}_c)$  includes the  $\mathcal{O}(\alpha_s)$  QCD corrections as well as the nonperturbative corrections up to  $\mathcal{O}(1/m_b^3)$

$$\kappa(\hat{m}_c) = 1 + \frac{\alpha_s(m_b)}{\pi} g(\hat{m}_c) + \frac{h_1(\hat{m}_c)}{2m_b^2} + \frac{h_2(\hat{m}_c)}{6m_b^3} \quad (24)$$

where

$$\begin{aligned} g(\hat{m}_c) &= \frac{A_0(\hat{m}_c)}{f(\hat{m}_c)} \\ h_1(\hat{m}_c) &= \lambda_1 + \frac{\lambda_2}{f(\hat{m}_c)} \left( -9 + 24\hat{m}_c^2 - 72\hat{m}_c^4 + 72\hat{m}_c^6 - 15\hat{m}_c^8 - 72\hat{m}_c^4 \log \hat{m}_c \right) \\ h_2(\hat{m}_c) &= \frac{\rho_1}{f(\hat{m}_c)} \left( 77 - 88\hat{m}_c^2 + 24\hat{m}_c^4 - 8\hat{m}_c^6 - 5\hat{m}_c^8 + 96 \log \hat{m}_c + 72\hat{m}_c^4 \log \hat{m}_c \right) \\ &\quad + \frac{\rho_2}{f(\hat{m}_c)} \left( 27 - 72\hat{m}_c^2 + 216\hat{m}_c^4 - 216\hat{m}_c^6 + 45\hat{m}_c^8 + 216\hat{m}_c^4 \log \hat{m}_c \right) \end{aligned} \quad (25)$$

The analytic expression for the perturbative function  $A_0(\hat{m}_c)$  can be found in [21].

### III. THE PARTIALLY INTEGRATED BRANCHING RATIO

An interesting experimentally accessible quantity is the dilepton invariant mass spectrum. Evaluating the  $\hat{u}$  integral in (16), and doing the integral over  $v \cdot \hat{q}$  by picking out the residues, we find for the dilepton invariant mass spectrum

$$\begin{aligned} \frac{d\mathcal{B}}{d\hat{s}} &= 2 \mathcal{B}_0 \left\{ \left[ \frac{1}{3} (1 - \hat{s})^2 (1 + 2\hat{s}) \left( 2 + \frac{\lambda_1}{m_b^2} \right) + \left( 1 - 15\hat{s}^2 + 10\hat{s}^3 \right) \left( \frac{\lambda_2}{m_b^2} - \frac{\rho_2}{m_b^3} \right) \right. \right. \\ &\quad \left. \left. - \frac{10\hat{s}^4 + 23\hat{s}^3 - 9\hat{s}^2 + 13\hat{s} + 11}{9(1 - \hat{s})} \frac{\rho_1}{m_b^3} \right] \left( |C_9^{\text{eff}}(\hat{s})|^2 + C_{10}^2 \right) \right. \\ &\quad + \left[ \frac{4}{3} (1 - \hat{s})^2 (2 + \hat{s}) \left( 2 + \frac{\lambda_1}{m_b^2} \right) + 4 \left( -6 - 3\hat{s} + 5\hat{s}^3 \right) \left( \frac{\lambda_2}{m_b^2} - \frac{\rho_2}{m_b^3} \right) \right. \\ &\quad \left. \left. - \frac{4(5\hat{s}^4 + 19\hat{s}^3 + 9\hat{s}^2 - 7\hat{s} + 22)}{9(1 - \hat{s})} \frac{\rho_1}{m_b^3} \right] \frac{|C_7^{\text{eff}}|^2}{\hat{s}} \right. \\ &\quad + \left[ 4(1 - \hat{s})^2 \left( 2 + \frac{\lambda_1}{m_b^2} \right) + 4 \left( -5 - 6\hat{s} + 7\hat{s}^2 \right) \left( \frac{\lambda_2}{m_b^2} - \frac{\rho_2}{m_b^3} \right) \right. \\ &\quad \left. \left. + \frac{4(3\hat{s}^3 - 17\hat{s}^2 + \hat{s} - 3)}{3(1 - \hat{s})} \frac{\rho_1}{m_b^3} \right] \text{Re}(C_9^{\text{eff}}(\hat{s})) C_7^{\text{eff}} \right. \\ &\quad \left. - \frac{16}{3} \frac{\rho_1}{m_b^3} \delta(1 - \hat{s}) \left( C_{10}^2 + \left( C_9^{\text{eff}}(\hat{s}) + 2C_7^{\text{eff}} \right)^2 \right) \right\}. \end{aligned} \quad (26)$$

The dependence on  $\mathcal{T}_{1-4}$  can be obtained by making the replacements (13) in (26). In this expression we have taken the limit  $\hat{m}_s \rightarrow 0$ . The corresponding expression with full  $\hat{m}_s$  dependence is given in Appendix B.

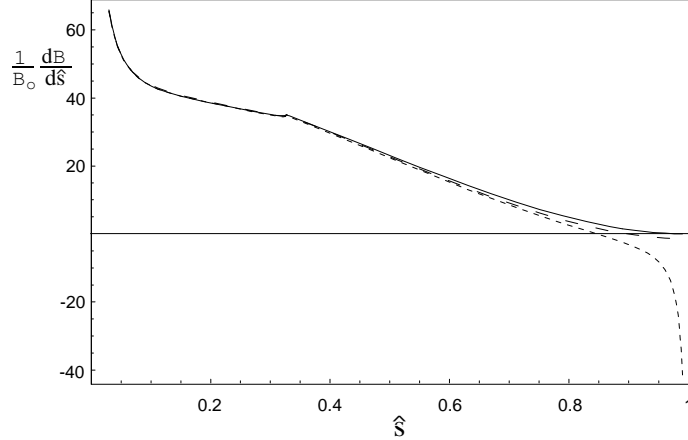


FIG. 1. The differential decay spectrum. The solid line shows the parton model prediction, the long-dashed line includes the  $\mathcal{O}(1/m_b^2)$  corrections and the short-dashed line contains all corrections up to  $\mathcal{O}(1/m_b^3)$ .

A plot of this distribution is shown in Fig. 1, where we have used the values for the nonperturbative matrix elements

$$\lambda_1 = -0.19 \text{ GeV}^2, \quad \lambda_2 = 0.12 \text{ GeV}^2. \quad (27)$$

For the matrix elements of the dimension six operators we use the generic size  $(\Lambda_{\text{QCD}})^3 \sim (0.5 \text{ GeV})^3$  as suggested by dimensional analysis. The vacuum saturation approximation [22] predicts  $\rho_1 > 0$ , as shown, and we find the displayed spectrum is fairly insensitive to the sign of the other dimension six matrix elements. One immediately notices divergences at both endpoints of this spectrum. The divergence at the  $\hat{s} \rightarrow 0$  endpoint is due to the intermediate photon going on-shell and is a well known feature of the decay  $B \rightarrow X_s \ell^+ \ell^-$  [1,9]. In this limit one expects this spectrum to reduce to the  $B \rightarrow X_s \gamma$  rate with an on-shell photon in the final state, convoluted with the fragmentation function giving the probability for a photon to fragment into a lepton pair. This correspondence is explicitly verified by the analytic form of the divergent term

$$\frac{1}{\mathcal{B}_0} \frac{d\mathcal{B}}{d\hat{s}} \Big|_{\hat{s} \rightarrow 0} \sim \frac{32 |C_7^{\text{eff}}|^2}{3 \hat{s}} \left( 1 + \frac{\lambda_1 - 9\lambda_2}{2m_b^2} - \frac{11\rho_1 - 27\rho_2}{6m_b^3} + \frac{\mathcal{T}_1 + 3\mathcal{T}_2 - 3(\mathcal{T}_3 + 3\mathcal{T}_4)}{2m_b^3} \right) \quad (28)$$

where the term multiplying  $1/\hat{s}$  is proportional to the total rate for  $B \rightarrow X_s \gamma$  [17]. As mentioned above, experimental cuts require us to stay away from this endpoint, and therefore automatically regulate this divergence.

The divergence at the  $\hat{s} \rightarrow 1$  endpoint is entirely due to the  $1/m_b^3$  operators as can be seen from Fig. 1. In this case the analytic form of the divergent term is

$$\frac{1}{\mathcal{B}_0} \frac{d\mathcal{B}}{d\hat{s}} \Big|_{\hat{s} \rightarrow 1} \sim \frac{32}{3m_b^3} \left( C_{10}^2 + (2C_7^{\text{eff}} + C_9^{\text{eff}}(\hat{s}))^2 \right) \frac{\rho_1}{1 - \hat{s}}. \quad (29)$$



This leads, upon integration, to an unphysical logarithmic divergence in the expression for the total rate that is regulated by the mass of the  $s$  quark. (Of course, it is only consistent to include the mass of the  $s$  quark in the upper limit of integration if one uses the spectrum with the full  $m_s$  dependence as given in Appendix B.) This divergence can be understood by considering a similar effect in the semileptonic decay  $B \rightarrow X_c \ell \bar{\nu}_\ell$  [23]. In that context, the origin of this divergence can be clarified by performing an OPE for the total, rather than the differential, rate [23,24]. Including dimension six operators in this OPE one finds a four fermion operator of the form

$$\frac{16\pi^2}{m_b^3} \bar{b} \gamma^\mu L c \bar{c} \gamma^\nu L b (g_{\mu\nu} - v_\mu v_\nu) \quad (30)$$

contributing to the rate. In [23] the matrix element of this operator was calculated at leading order in perturbation theory by integrating out the  $c$  quark and its contribution to the total rate was found to be  $\rho_1 \log(\hat{m}_c)$ . To calculate this matrix element it was essential that the mass of the  $c$  quark be large compared to the QCD scale  $\Lambda_{QCD}$ . Consequently, for the decay  $B \rightarrow X_s \ell^+ \ell^-$  where the same operator with  $s$  quarks rather than  $c$  quarks appears, these methods are not applicable because the  $s$  quark is too light. Including higher orders in perturbation theory the matrix element of the four fermion operator contains logarithms of the form  $\alpha_s^n \log^{n+1}(\hat{m}_s)$  which are of order unity, making a perturbative calculation of this matrix element impossible. Thus, a seventh non-perturbative matrix element  $f_1$  defined in (15) is required. It contributes only at the  $\hat{s} \rightarrow 1$  endpoint of the spectrum and cancels the logarithmic divergence proportional to  $\rho_1 \log(\hat{m}_s)$  in the total rate

$$\frac{d\mathcal{B}}{d\hat{s}} \rightarrow \frac{d\mathcal{B}}{d\hat{s}} - \frac{32}{3m_b^3} \mathcal{B}_0 \left( C_{10}^2 + (2C_7^{\text{eff}} + C_9^{\text{eff}}(\hat{s}))^2 \right) \delta(1 - \hat{s}) (\rho_1 \log(\hat{m}_s) - f_1). \quad (31)$$

Another noticeable feature in the dilepton invariant mass spectrum is the cusp due to the  $c\bar{c}$  threshold. Near this value of  $\hat{s}$  the methods we used to calculate the physical spectrum fail because of long distance contributions from the resonant decay  $B \rightarrow X_s J/\psi$ , where the  $J/\psi$  subsequently decays into two leptons. Experimentally one deals with this resonance region by simply cutting it out. Thus, to compare reliably to experiment we should include such a cut in our calculation. Defining the partially integrated branching ratio by

$$\mathcal{B}_\chi = \frac{1}{\mathcal{B}_0} \int_\chi^1 d\hat{s} \frac{d\mathcal{B}}{d\hat{s}} \quad (32)$$

we plot the contribution of the individual matrix elements relative to the leading order parton result in Fig. 2. For the generic size  $\rho_i \sim (\Lambda_{QCD}^3)$  used in this plot, the contribution from  $\rho_1$  is of the same size as the contribution from dimension five operators. This implies that including the  $\mathcal{O}(1/m_b^2)$  corrections for this decay does not significantly decrease the nonperturbative uncertainties. We see that the nonperturbative contributions become more dominant as the accessible phase space is decreased<sup>2</sup>. For  $\chi \sim .75$  the uncertainty from

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<sup>2</sup>We emphasize that the sizes of the  $\rho_i$  contributions shown here should not be taken as accurate indications of the actual size of the corrections, but rather as estimates of the uncertainty in the prediction.

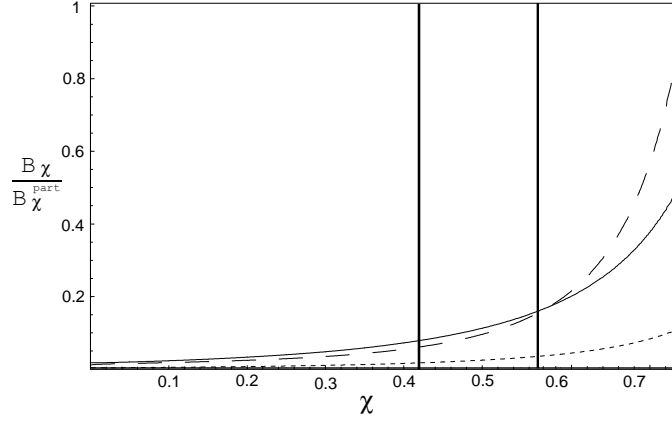


FIG. 2. The fractional contributions to  $B_\chi$  with respect to the parton model result from the  $\mathcal{O}(1/m_b^3)$  operators. The solid, dashed and dotted lines correspond to the contributions from  $\lambda_2$ ,  $\rho_1$  and  $\rho_2$ , respectively. The contribution from  $\lambda_1$  is too small to be seen. The two vertical lines illustrate the positions of the  $J/\psi$  and the  $\psi'$  resonance.

the  $\rho_1$  matrix element is of the same size as the parton model prediction. This is a clear signal that the OPE is no longer valid if the phase space is restricted to be too close to the endpoint  $\hat{s} = 1$ . This breakdown of the OPE close to the endpoint is a well-known feature encountered in this approach to the study of inclusive decays [26]. Unfortunately, in this endpoint region a shape function does not exist and an alternate approach, such as heavy hadron chiral perturbation theory for exclusive final states, must be used [25].

A cut of  $\chi = (14.33 \text{ GeV}^2/m_b^2) = 0.59$  has been suggested by the CLEO collaboration in order to eliminate the resonance region [8]. For this value the partially integrated rate is

$$\begin{aligned} \mathcal{B}_{0.59} = & 3.8 + 1.9 \left( \frac{\lambda_1}{m_b^2} + \frac{\mathcal{T}_1 + 3\mathcal{T}_2}{m_b^3} \right) - 134.7 \left( \frac{\lambda_2}{m_b^2} + \frac{\mathcal{T}_1 + 3\mathcal{T}_2}{3m_b^3} \right) \\ & + 614.9 \frac{\rho_1}{m_b^3} + 134.7 \frac{\rho_2}{m_b^3} + 560.2 \frac{f_1}{m_b^3}. \end{aligned} \quad (33)$$

At this value of the cut  $\chi$  the coefficients of the nonperturbative matrix elements clearly indicate a poorly converging OPE. One can estimate the uncertainty induced by the  $\mathcal{O}(1/m_b^3)$  parameters by fixing  $\lambda_i$  to the values given in (27), then randomly varying the magnitudes of the parameters  $\rho_i, \mathcal{T}_i$  and  $f_1$  between  $-(0.5 \text{ GeV})^3$  and  $(0.5 \text{ GeV})^3$  as suggested by dimensional analysis. We also impose positivity of  $\rho_1$  as indicated by the vacuum saturation approximation [22], and we enforce the constraint

$$\rho_2 - \mathcal{T}_2 - \mathcal{T}_4 = \frac{\left( \frac{\alpha_s(m_c)}{\alpha_s(m_b)} \right)^{3/\beta_0} M_B^2 \Delta M_B (M_D + \bar{\Lambda}) - M_D^2 \Delta M_D (M_B + \bar{\Lambda})}{M_B + \bar{\Lambda} - \left( \frac{\alpha_s(m_c)}{\alpha_s(m_b)} \right)^{3/\beta_0} (M_D + \bar{\Lambda})} \quad (34)$$

derived from the ground state meson mass splittings  $\Delta M_H = M_{H^*} - M_H$  ( $H = B, D$ ) [18]. Here  $\beta_0$  is the usual coefficient of the beta function  $\beta_0 = 11 - 2/3n_f = 25/3$  for 4 light flavours. Taking the  $1\sigma$  deviation as a reasonable estimate of the uncertainties from

$\mathcal{O}(1/m_b^3)$  contributions, we find the uncertainty in  $\mathcal{B}_{0.59}$  to be at the 10% level. It is clear from (33) that the  $\rho_1$  contribution is large, and relaxing the positivity constraint on  $\rho_1$  enlarges the uncertainty to about 20%. A similar statement can be made regarding the  $D\bar{O}$  analysis [8] where the phase space cut is slightly higher, and the nonperturbative corrections are correspondingly somewhat larger. Since the cut on  $\hat{s}$  cannot be lowered because of the  $\psi'$  resonance, these uncertainties are intrinsic to our approach in the large dilepton invariant mass region.

It is important to notice that in the invariant mass region below the  $J/\psi$  resonance, the uncertainties from these matrix elements are much smaller. For example, integrating the differential spectrum up to the cut specified in the CLEO analysis [5]  $\hat{s} = (M_{J/\psi} - 0.1\text{GeV})^2/m_b^2 = 0.35$  we find

$$\int_{0.01}^{0.35} d\hat{s} \frac{d\mathcal{B}}{d\hat{s}} = 22.0 \left( 1 + 0.5 \left( \frac{\lambda_1}{m_b^2} + \frac{\mathcal{T}_1 + 3\mathcal{T}_2}{m_b^3} \right) + 1.2 \left( \frac{\lambda_2}{m_b^2} + \frac{\mathcal{T}_2 + 3\mathcal{T}_4}{3m_b^3} \right) - 3.7 \frac{\rho_1}{m_b^3} - 1.2 \frac{\rho_2}{m_b^3} \right). \quad (35)$$

It is still true that the coefficient of the  $\rho_1$  term is  $\sim 10$  times larger than that of the  $\lambda_1$  term, but both sets of nonperturbative corrections are small relative to the parton level result in this region. Although this does not allow us to draw a strong conclusion about the convergence of the OPE, we can conclude that in this region the  $\mathcal{O}(1/m_b^3)$  nonperturbative corrections are not a significant source of theoretical uncertainty.

#### IV. THE FORWARD-BACKWARD ASYMMETRY

The differential forward-backward asymmetry is defined by

$$\frac{d\mathcal{A}}{d\hat{s}} = \int_0^1 dz \frac{d\mathcal{B}}{dz d\hat{s}} - \int_{-1}^0 dz \frac{d\mathcal{B}}{dz d\hat{s}} \quad (36)$$

where

$$z = \cos\theta = \frac{\hat{u}}{\hat{u}(\hat{s}, \hat{m}_s)} \quad (37)$$

parameterizes the angle between the  $b$  quark and the  $\ell^+$  in the dilepton CM frame. It has been shown [4] that new physics can modify this spectrum, so it is interesting to see how  $\mathcal{O}(1/m_b^3)$  terms contribute to the SM prediction.

Integrating the triple differential decay rate (16) we find

$$\begin{aligned} \frac{d\mathcal{A}}{d\hat{s}} = & C_7^{\text{eff}} C_{10} \left( -8(1 - \hat{s})^2 - \frac{4(3 + 2\hat{s} + 3\hat{s}^2) \lambda_1}{3m_b^2} + \frac{4(7 + 10\hat{s} - 9\hat{s}^2) \lambda_2}{m_b^2} \right. \\ & \left. + \frac{4(5 + 2\hat{s} + \hat{s}^2) \rho_1}{3m_b^3} - \frac{4(7 + 10\hat{s} - 9\hat{s}^2) \rho_2}{m_b^3} \right) \\ & + C_9^{\text{eff}}(\hat{s}) C_{10} \left( -4\hat{s}(1 - \hat{s})^2 - \frac{2\hat{s}(3 + 2\hat{s} + 3\hat{s}^2) \lambda_1}{3m_b^2} + \frac{2\hat{s}(9 + 14\hat{s} - 15\hat{s}^2) \lambda_2}{m_b^2} \right. \\ & \left. - \frac{2\hat{s}(1 + 2\hat{s} + 5\hat{s}^2) \rho_1}{3m_b^3} - \frac{2\hat{s}(1 + 6\hat{s} - 15\hat{s}^2) \rho_2}{m_b^3} \right) \end{aligned} \quad (38)$$

Here we have again omitted the trivial dependence on  $\mathcal{T}_{1-4}$ .

It is clear from this expression that the third order terms do not have abnormally large coefficients, and therefore introduce only small variations relative to the second order expression.

An experimentally more useful quantity is the normalized FB asymmetry defined by

$$\frac{d\bar{\mathcal{A}}}{d\hat{s}} = \frac{d\mathcal{A}}{d\hat{s}} / \frac{d\mathcal{B}}{d\hat{s}}. \quad (39)$$

Unfortunately, this normalized asymmetry has inherited the poor behavior of the differential branching ratio in the endpoint region. In Fig. 3 we illustrate the uncertainties of the

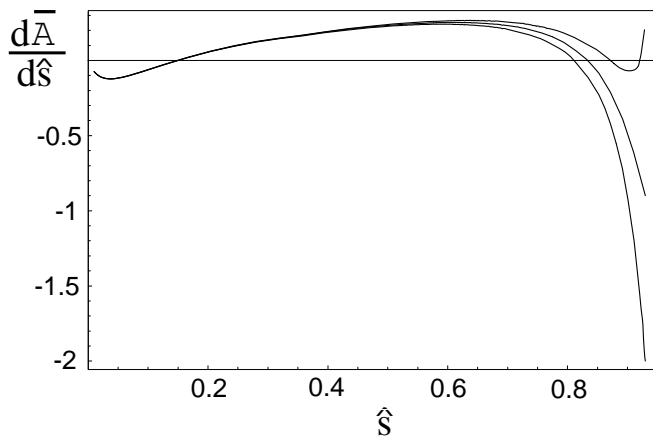


FIG. 3. The normalized forward backward asymmetry. The three curves show the mean value and the  $1\sigma$  uncertainty of the forward backward asymmetry, obtained in a way similar to that explained in section III.

normalized FB asymmetry originating from the matrix elements of the dimension six operators. The three curves show the mean value and the  $1\sigma$  uncertainty of the forward backward asymmetry, obtained in a way similar to that explained in section III. We can see that up to a value of  $\hat{s} = 0.7$  the uncertainties are small, but for larger values of the dilepton invariant mass the uncertainties increase rapidly. Because of the necessity of the cut to eliminate the  $c\bar{c}$  resonances, the accessible high dilepton invariant mass region is therefore restricted to a few hundred MeV. For the dilepton invariant mass region below the  $J/\psi$  resonance ( $\hat{s} < 0.35$ ) the uncertainties are small.

## V. EXTRACTING $\bar{\Lambda}$ AND $\lambda_1$ FROM THE HADRON INVARIANT MASS MOMENTS

Throughout this paper we have fixed the values of the leading non-perturbative parameters  $\bar{\Lambda}$ ,  $\lambda_1$ , and  $\lambda_2$ . However, these values must be determined from experiment. The most sensitive observables for this purpose are those which vanish in the parton model. It is interesting to ask how severely our ignorance of the values of the  $\mathcal{O}(1/m_b^3)$  parameters compromises our ability to extract the values of  $\bar{\Lambda}$  and  $\lambda_1$  from such a measurement. It

has been suggested by Ali and Hiller [10] that one use the first two moments of the hadron invariant mass spectrum defined by

$$\langle S_H^n \rangle = \int (S_H - M_H^2)^n \frac{d\mathcal{B}}{dS_H} dS_H. \quad (40)$$

This idea is similar to the approaches used in the semileptonic  $B \rightarrow X_c \ell \nu_\ell$  [18] and the rare radiative  $B \rightarrow X_s \gamma$  decays [17], though the experimental task is more difficult in this case due to the small size of the branching ratio. To calculate these hadronic moments we relate them to calculable partonic moments via

$$\begin{aligned} \langle S_H \rangle &= \bar{\Lambda}^2 - \frac{\bar{\Lambda}(\lambda_1 + 3\lambda_2)}{M_B} \\ &+ \left( M_B^2 - 2M_B \bar{\Lambda} + \bar{\Lambda}^2 + \lambda_1 + 3\lambda_2 - \frac{\rho_1 + 3\rho_2}{2M_B} + \frac{\mathcal{T}_1 + \mathcal{T}_3 + 3(\mathcal{T}_2 + \mathcal{T}_4)}{2M_B} \right) \langle \hat{s}_0 \rangle \\ &+ \left( 2M_B \bar{\Lambda} - 2\bar{\Lambda}^2 - \lambda_1 - 3\lambda_2 + \frac{\bar{\Lambda}(\lambda_1 + 3\lambda_2)}{M_B} + \frac{\rho_1 + 3\rho_2}{2M_B} \right. \\ &\quad \left. - \frac{\mathcal{T}_1 + \mathcal{T}_3 + 3(\mathcal{T}_2 + \mathcal{T}_4)}{2M_B} \right) \langle x_0 \rangle \end{aligned} \quad (41)$$

$$\begin{aligned} \langle S_H^2 \rangle &= \left( M_B^4 - 4M_B^3 \bar{\Lambda} + 6M_B^2 \bar{\Lambda}^2 + 2M_B^2(\lambda_1 + 3\lambda_2) - 4M_B \bar{\Lambda}^3 - 4M_B \bar{\Lambda}(\lambda_1 + 3\lambda_2) \right. \\ &\quad \left. - M_B(\rho_1 + 3\rho_2) + M_B(\mathcal{T}_1 + \mathcal{T}_3 + 3(\mathcal{T}_2 + \mathcal{T}_4)) \right) \langle \hat{s}_0^2 \rangle \\ &+ 4 \left( M_B^2 \bar{\Lambda}^2 - 2M_B \bar{\Lambda}^3 - M_B \bar{\Lambda}(\lambda_1 + 3\lambda_2) \right) \langle x_0^2 \rangle \\ &+ \left( 4M_B^3 \bar{\Lambda} - 12M_B^2 \bar{\Lambda}^2 - 2M_B^2(\lambda_1 + 3\lambda_2) + 12M_B \bar{\Lambda}^3 + 10M_B \bar{\Lambda}(\lambda_1 + 3\lambda_2) \right. \\ &\quad \left. + M_B(\rho_1 + 3\rho_2) - M_B(\mathcal{T}_1 + \mathcal{T}_3 + 3(\mathcal{T}_2 + \mathcal{T}_4)) \right) \langle x_0 \hat{s}_0 \rangle \\ &+ 2 \left( M_B^2 \bar{\Lambda}^2 - 2M_B \bar{\Lambda}^3 - (\lambda_1 + 3\lambda_2) M_B \bar{\Lambda} \right) \langle \hat{s}_0 \rangle \\ &+ 4M_B \bar{\Lambda}^3 \langle x_0 \rangle \end{aligned} \quad (42)$$

where we have used the mass relation

$$m_b = M_B - \bar{\Lambda} + \frac{\lambda_1 + 3\lambda_2}{2m_b} - \frac{\rho_1 + 3\rho_2}{4m_b^2} + \frac{\mathcal{T}_1 + \mathcal{T}_3 + 3(\mathcal{T}_2 + \mathcal{T}_4)}{4m_b^2} \quad (43)$$

appropriate at this order in the OPE. We therefore have to calculate the first two moments of the parton energy  $\langle x_0 \rangle, \langle x_0^2 \rangle$  and parton invariant mass  $\langle \hat{s}_0 \rangle, \langle \hat{s}_0^2 \rangle$ , as well as the mixed moment  $\langle x_0 \hat{s}_0 \rangle$ . Defining

$$M^{(m,n)} = \langle x_0^m \hat{s}_0^n \rangle = \frac{1}{\mathcal{B}_0} \int_{\hat{m}_s}^{\frac{1}{2}(1-\chi)} dx_0 \int_{\hat{m}_s^2}^{x_0^2} d\hat{s}_0 x_0^m \hat{s}_0^n \frac{d^2\mathcal{B}}{dx_0 d\hat{s}_0}, \quad (44)$$

we give the results for the required partonic moments in Appendix C. As before, we have included the dependence on the cut on the lepton invariant mass in these results. It is important to note that the results for the partonic moments given in Appendix C are expressed in terms of the  $b$  quark mass  $m_b$  and must be re-expressed in terms of the  $B$  meson mass  $M_B$  using the mass relation (43). Using again for the cut on the invariant mass the value proposed by CLEO  $q^2 > 14.33 \text{ GeV}^2$ , we find for the two moments

$$\langle S_H \rangle = M_B^2 \left[ 0.36 \frac{\bar{\Lambda}}{M_B} + 0.64 \frac{\lambda_1}{M_B^2} + 0.67 \frac{\lambda_2}{M_B^2} - 0.09 \frac{\bar{\Lambda}^2}{M_B^2} + 8.48 \frac{\rho_1}{M_B^3} + 3.79 \frac{\rho_2}{M_B^3} \right. \\ \left. + 1.09 \frac{\bar{\Lambda}\lambda_1}{M_B^3} + 4.88 \frac{\bar{\Lambda}\lambda_2}{M_B^3} - 0.41 \frac{\bar{\Lambda}^3}{M_B^3} + 0.73 \frac{\mathcal{T}_1 + 3\mathcal{T}_2}{M_B^3} + 0.31 \frac{\mathcal{T}_3 + 3\mathcal{T}_4}{M_B^3} \right] \quad (45)$$

$$\langle S_H^2 \rangle = M_B^4 \left[ -0.05 \frac{\lambda_1}{M_B^2} + 0.14 \frac{\bar{\Lambda}^2}{M_B^2} - 0.53 \frac{\rho_1}{M_B^3} - 0.21 \frac{\rho_2}{M_B^3} + 0.63 \frac{\bar{\Lambda}\lambda_1}{M_B^3} + 0.46 \frac{\bar{\Lambda}\lambda_2}{M_B^3} \right. \\ \left. - 0.05 \frac{\mathcal{T}_1 + 3\mathcal{T}_2}{M_B^3} \right] \quad (46)$$

Consider first the expression for  $\langle S_H^2 \rangle$ . The  $\lambda_1$  term has a small coefficient and tends to cancel against higher order corrections, making this moment particularly insensitive to  $\lambda_1$ . We can see the problem another way by solving this equation for  $\lambda_1$ : the solution exhibits a pole near  $\bar{\Lambda} = 0.4$ , close to the expected value of  $\bar{\Lambda}$  [12]. As a result, the extracted value of  $\lambda_1$  is extremely sensitive to the values of the higher order parameters. Since the presence of this pole persists as the value of the cut is changed, we conclude that this observable is unsuitable for extracting  $\lambda_1$ .

For the first moment  $\langle S_H \rangle$  the convergence of the OPE is much better. Estimating the uncertainties from the unknown values of the dimension six operators by the method explained in section III, we present the resulting constraint in the  $\bar{\Lambda} - \lambda_1$  plane in Fig. 4. Superimposed on this figure is the ellipse obtained in [27] from an analogous study of mo-

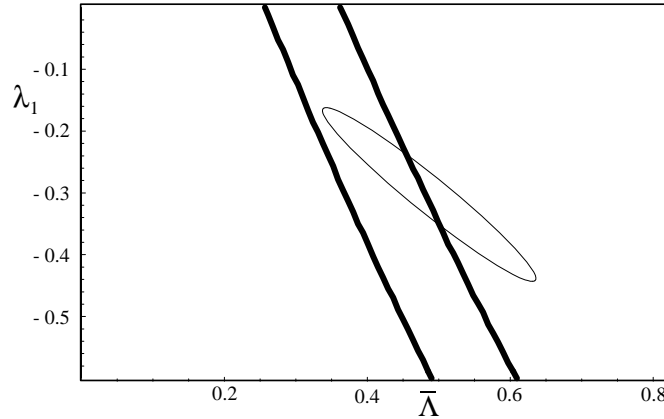


FIG. 4. The constraint in the  $\bar{\Lambda} - \lambda_1$  plane from  $\langle S_H \rangle$ . The width of the band is entirely due to uncertainties from  $\mathcal{O}(1/m_b^3)$  operators. The ellipse is the equivalent constraint from moments of semileptonic  $B \rightarrow X_c \ell \nu_\ell$ . It is only the relative orientation, and not location, of the constraints which has meaning.

ments of  $B \rightarrow X_c \ell \nu_\ell$ . Unfortunately, the bound from our analysis is nearly parallel to the major axis of this ellipse and, since it is only the relative orientation of the constraints which has meaning in this figure, this moment does not provide much additional information about the values of  $\bar{\Lambda}$  or  $\lambda_1$ .

## VI. CONCLUSIONS

Our purpose in this paper has been to study nonperturbative uncertainties in the rare inclusive decay  $B \rightarrow X_s \ell^+ \ell^-$ . Building on previous studies which evaluated the leading nonperturbative corrections, we have parameterized the corrections arising at  $\mathcal{O}(1/m_b^3)$  in terms of two matrix elements of local operators  $\rho_{1,2}$ , four matrix elements of non local operators  $\mathcal{T}_{1-4}$ , and one matrix element of a four fermion operator  $f_1$ .

The numerical values of these parameters are unknown, yet even so a knowledge of the analytic form of the corrections allows us to study the convergence properties of the operator product expansion in various regions of phase space. Furthermore, the assumption that these parameters, being of nonperturbative origin, should be  $\mathcal{O}(\Lambda_{\text{QCD}}^3)$  permits us to make numerical estimates of theoretical uncertainties in observable quantities.

We first considered the corrections to the differential spectrum  $d\mathcal{B}/d\hat{s}$ . The experimental spectrum contains two prominent resonances due to intermediate  $J/\psi$  and  $\psi'$  production, and the necessity of cutting these resonances out divides the accessible spectrum into two parts: the region of low dilepton invariant mass below the  $J/\psi$  resonance, and the region of high dilepton invariant mass above the  $\psi'$  resonance. In the first region, we find that the parton level calculation dominates, and that nonperturbative corrections are small. The operator product expansion appears to be converging according to expectation and, should it be possible to take experimental data in this region, the results will not suffer from significant nonperturbative uncertainties.

On the other hand we do find that, as expected, the nonperturbative uncertainties increase as one moves into the high dilepton invariant mass region. It is well known that as one approaches this endpoint, the operator product expansion breaks down. The interesting result of our study, however, is that the expansion breaks down somewhat earlier than was anticipated, with the  $\mathcal{O}(1/m_b^3)$  uncertainties coming to dominate the integrated rate once the available range of  $\hat{s}$  is reduced to about one quarter of the full range. We also showed that the rate obtained by integrating over the entire region above the  $\psi'$  resonance contains uncertainties from dimension six matrix elements at the 10% level, the uncertainties being dominated by those from the  $\rho_1$  matrix element. This result may impact the potential for doing precise searches for new physics using data from this region of phase space.

We also studied the contributions from dimension six operators to the forward-backward asymmetry. This quantity probes different combinations of Wilson coefficients than the rate, and has been proposed as a complimentary source of information about possible new physics effects. As with the rate, the spectrum  $d\mathcal{A}/d\hat{s}$  is divided into two regions by the  $J/\psi$  and  $\psi'$  resonances. We find that the dimension six contributions are not unduly large anywhere in the phase space, suggesting that this observable has a well behaved OPE. However if the differential forward-backward asymmetry is normalized to the differential rate, the resulting spectrum contains large uncertainties in the high dilepton invariant mass region. In the region of phase space below the  $J/\psi$  resonance, however, we find the nonperturbative corrections to be small.

Finally we addressed a recent proposal suggesting that hadron invariant mass moments of the differential spectrum for  $B \rightarrow X_s \ell^+ \ell^-$  could, due to the sensitivity of these moments to nonperturbative effects, be used to constrain the values of the HQET parameters  $\lambda_1$  and  $\bar{\Lambda}$ . In the low  $S_H$  region the moments are suppressed relative to the rate and, considering the

already tiny branching ratio, it is unlikely that experimental measurements in this region will be forthcoming. Therefore we focused our attention on the high invariant mass region. In this region, we found that the first of these moments  $\langle S_H \rangle$  provided a constraint in the  $\bar{\Lambda} - \lambda_1$  plane, but that this constraint was nearly the same as those derived from other, more experimentally promising, processes, and therefore seems to be of limited interest for this purpose. As for the second invariant mass moment  $\langle S_H^2 \rangle$ , we found that the nonperturbative uncertainties were such that it was not possible to extract a stable constraint on the values of  $\bar{\Lambda}$  or  $\lambda_1$ . From these results, we conclude that these moments are not well suited to the extraction of these parameters.

## VII. ACKNOWLEDGEMENTS

We would like to thank Michael Luke for helpful discussions. This work was supported in part by the National Science and Engineering Research Council of Canada.

## APPENDIX A: THE CONTRIBUTION OF DIMENSION SIX OPERATORS TO THE FORM FACTORS

In this appendix we present the form factors  $T_i$ . These form factors have been calculated previously up to  $\mathcal{O}(1/m_b^2)$  [9], and we do not reproduce those results here. We decompose the new contributions arising at  $\mathcal{O}(1/m_b^3)$  as

$$T_i^{L/R} = 2M_B \left( T_i^{(C_9 \pm C_{10})^2} \left( C_{10} \pm C_9^{\text{eff}} \right)^2 + T_i^{C_7^2} |C_7^{\text{eff}}|^2 + T_i^{C_7(C_9 \pm C_{10})} C_7^{\text{eff}} (C_9 \pm C_{10}) \right). \quad (\text{A1})$$

For completeness we have included the full  $\hat{m}_s$  dependence in these expressions, though in our analysis we set  $\hat{m}_s = 0$ . Defining  $x = 1 - 2v \cdot \hat{q} + \hat{s} + i\epsilon$  with  $\hat{q} = \frac{q}{m_b}$ ,  $\hat{s} = \hat{q}^2$  and  $\hat{\rho}_i = \rho_i/m_b^3$ , we find that the third order contributions are

$$\begin{aligned} T_1^{(C_9 \pm C_{10})^2} &= \frac{1}{12x} (\hat{\rho}_1 + 3\hat{\rho}_2) \\ &\quad - \frac{1}{6x^2} \left( (2 + \hat{s}) \hat{\rho}_1 - 3(2 - \hat{s}) \hat{\rho}_2 + (\hat{\rho}_1 + 3\hat{\rho}_2) v \cdot \hat{q} - (\hat{\rho}_1 + 3\hat{\rho}_2) (v \cdot \hat{q})^2 \right) \\ &\quad - \frac{2}{3x^3} (\hat{\rho}_1 + 3\hat{\rho}_2) (1 - v \cdot \hat{q}) (\hat{s} - (v \cdot \hat{q})^2) \\ &\quad + \frac{4}{3x^4} \hat{\rho}_1 (1 - v \cdot \hat{q})^2 (\hat{s} - (v \cdot \hat{q})^2) \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} T_2^{(C_9 \pm C_{10})^2} &= -\frac{1}{6x} (\hat{\rho}_1 + 3\hat{\rho}_2) \\ &\quad - \frac{1}{3x^2} (4\hat{\rho}_1 + 6\hat{\rho}_2 - (\hat{\rho}_1 + 3\hat{\rho}_2) v \cdot \hat{q}) \\ &\quad + \frac{2}{3x^3} \left( 3\hat{s}\hat{\rho}_2 - 2(2\hat{\rho}_1 + 3\hat{\rho}_2) v \cdot \hat{q} + 2(2\hat{\rho}_1 + 3\hat{\rho}_2) (v \cdot \hat{q})^2 \right) \\ &\quad + \frac{8}{3x^4} \hat{\rho}_1 (1 - v \cdot \hat{q}) (\hat{s} - (v \cdot \hat{q})^2) \end{aligned} \quad (\text{A3})$$



$$\begin{aligned}
T_3^{(C_9 \pm C_{10})^2} &= -\frac{1}{6x^2} (\hat{\rho}_1 + 3\hat{\rho}_2) v \cdot \hat{q} \\
&+ \frac{2}{3x^3} (1 - v \cdot \hat{q}) (3\hat{\rho}_2 - (\hat{\rho}_1 + 3\hat{\rho}_2) v \cdot \hat{q}) \\
&+ \frac{4}{3x^4} \hat{\rho}_1 (1 - v \cdot \hat{q}) (\hat{s} - (v \cdot \hat{q})^2)
\end{aligned} \tag{A4}$$

$$\begin{aligned}
T_1^{C_7^2} &= \frac{1}{3\hat{s}^2 x} \left( (\hat{\rho}_1 + 3\hat{\rho}_2) \left( (1 - 5\hat{m}_s^2) \hat{s} - 2(1 + \hat{m}_s^2) (v \cdot \hat{q})^2 \right) \right. \\
&- \frac{2}{3\hat{s}^2 x^2} \left( \hat{s} \left( (4\hat{m}_s^2 + \hat{s} + \hat{m}_s^2 \hat{s} - 4) \hat{\rho}_1 + 3(8\hat{m}_s^2 + \hat{s} + \hat{m}_s^2 \hat{s}) \hat{\rho}_2 \right) \right. \\
&\quad \left. + (1 - 5\hat{m}_s^2) \hat{s} (\hat{\rho}_1 + 3\hat{\rho}_2) v \cdot \hat{q} + (1 + \hat{m}_s^2) ((8 - \hat{s}) \hat{\rho}_1 - 3\hat{s} \hat{\rho}_2) (v \cdot \hat{q})^2 \right. \\
&\quad \left. - 2(1 + \hat{m}_s^2) (\hat{\rho}_1 + 3\hat{\rho}_2) (v \cdot \hat{q})^3 \right) \\
&- \frac{8}{3\hat{s}^2 x^3} (1 + \hat{m}_s^2) (1 - v \cdot \hat{q}) (\hat{s} (\hat{\rho}_1 + 3\hat{\rho}_2) - 2(2\hat{\rho}_1 + 3\hat{\rho}_2) v \cdot \hat{q}) (\hat{s} - (v \cdot \hat{q})^2) \\
&- \frac{16}{3\hat{s}^2 x^4} \hat{\rho}_1 (1 - v \cdot \hat{q}) (\hat{s} - (v \cdot \hat{q})^2) \\
&\quad \left( \hat{s} + 3\hat{m}_s^2 \hat{s} + (1 + \hat{m}_s^2) \hat{s} v \cdot \hat{q} - 2(1 + \hat{m}_s^2) (v \cdot \hat{q})^2 \right)
\end{aligned} \tag{A5}$$

$$\begin{aligned}
T_2^{C_7^2} &= \frac{2}{3\hat{s}x} (1 + \hat{m}_s^2) (\hat{\rho}_1 + 3\hat{\rho}_2) \\
&+ \frac{4}{3\hat{s}x^2} (1 + \hat{m}_s^2) (4\hat{\rho}_1 - (\hat{\rho}_1 + 3\hat{\rho}_2) v \cdot \hat{q}) \\
&+ \frac{8}{3\hat{s}x^3} (1 + \hat{m}_s^2) (3\hat{s}\hat{\rho}_2 + 2(2\hat{\rho}_1 + 3\hat{\rho}_2) v \cdot \hat{q} - 2(2\hat{\rho}_1 + 3\hat{\rho}_2) (v \cdot \hat{q})^2) \\
&- \frac{32}{3\hat{s}x^4} (1 + \hat{m}_s^2) \hat{\rho}_1 (1 - v \cdot \hat{q}) (\hat{s} - (v \cdot \hat{q})^2)
\end{aligned} \tag{A6}$$

$$\begin{aligned}
T_3^{C_7^2} &= -\frac{2}{3\hat{s}^2 x} (1 - \hat{m}_s^2) (\hat{\rho}_1 + 3\hat{\rho}_2) v \cdot \hat{q} \\
&- \frac{2}{3\hat{s}^2 x^2} (1 - \hat{m}_s^2) v \cdot \hat{q} ((8 - \hat{s}) \hat{\rho}_1 - 3\hat{s}\hat{\rho}_2 - 2(\hat{\rho}_1 + 3\hat{\rho}_2) v \cdot \hat{q}) \\
&+ \frac{8}{3\hat{s}^2 x^3} (1 - \hat{m}_s^2) (1 - v \cdot \hat{q}) \\
&\quad \left( \hat{s} (2\hat{\rho}_1 + 3\hat{\rho}_2) + \hat{s} (\hat{\rho}_1 + 3\hat{\rho}_2) v \cdot \hat{q} - 2(2\hat{\rho}_1 + 3\hat{\rho}_2) (v \cdot \hat{q})^2 \right) \\
&- \frac{16}{3\hat{s}^2 x^4} (1 - \hat{m}_s^2) (1 - v \cdot \hat{q}) \hat{\rho}_1 (\hat{s}^2 - 2\hat{s} v \cdot \hat{q} - \hat{s}(v \cdot \hat{q})^2 + 2(v \cdot \hat{q})^3)
\end{aligned} \tag{A7}$$

$$\begin{aligned}
T_1^{C_7(C_9 \pm C_{10})} &= -\frac{1}{x} (\hat{\rho}_1 + 3\hat{\rho}_2) \\
&- \frac{1}{3\hat{s}x^2} \left( 2(3 + \hat{m}_s^2) \hat{s} (\hat{\rho}_1 + 3\hat{\rho}_2) - 6(4\hat{\rho}_2 + \hat{s}(\hat{\rho}_1 + 3\hat{\rho}_2)) v \cdot \hat{q} \right. \\
&\quad \left. + 2(3 - \hat{m}_s^2) (\hat{\rho}_1 + 3\hat{\rho}_2) (v \cdot \hat{q})^2 \right) \\
&+ \frac{8}{3\hat{s}x^3} \left( (1 - \hat{m}_s^2) \hat{\rho}_1 - 3\hat{m}_s^2 \hat{\rho}_2 \right) (1 - v \cdot \hat{q}) (\hat{s} - (v \cdot \hat{q})^2) \\
&- \frac{16}{3\hat{s}x^4} \hat{\rho}_1 (1 - v \cdot \hat{q}) (\hat{s}^2 - (1 - \hat{m}_s^2) \hat{s} v \cdot \hat{q} - \hat{s}(v \cdot \hat{q})^2 \\
&\quad + (1 - \hat{m}_s^2) (v \cdot \hat{q})^3)
\end{aligned} \tag{A8}$$

$$T_2^{C_7(C_9 \pm C_{10})} = \frac{4}{3x^2} (\hat{\rho}_1 + 3\hat{\rho}_2) + \frac{8}{x^3} \hat{\rho}_2 (\hat{m}_s^2 + v \cdot \hat{q}) \quad (\text{A9})$$

$$\begin{aligned} T_3^{C_7(C_9 \pm C_{10})} &= \frac{2}{3\hat{s}x^2} \left( 12\hat{\rho}_2 - (3 + \hat{m}_s^2) (\hat{\rho}_1 + 3\hat{\rho}_2) v \cdot \hat{q} \right) \\ &\quad - \frac{8}{3\hat{s}x^3} \left( \hat{\rho}_1 + \hat{m}_s^2 \hat{\rho}_1 + 3\hat{m}_s^2 \hat{\rho}_2 \right) (1 - v \cdot \hat{q}) v \cdot \hat{q} \\ &\quad + \frac{16}{3\hat{s}x^4} \left( 1 + \hat{m}_s^2 \right) \hat{\rho}_1 (1 - v \cdot \hat{q}) \left( \hat{s} - (v \cdot \hat{q})^2 \right) \end{aligned} \quad (\text{A10})$$

## APPENDIX B: THE DILEPTON INVARIANT MASS SPECTRUM WITH FULL MASS DEPENDENCE

In this Appendix we present the dilepton invariant mass spectrum for a finite  $s$ -quark mass  $m_s$ . The spectrum originating from operators of dimension  $d \leq 5$  has been presented in Eq. (47) of [9]. The contributions from the time-ordered operators  $\mathcal{T}_{1-4}$  can be obtained by making the replacement (13) in this equation. Since the dilepton invariant mass distribution is independent of the definition of the four velocity of the heavy quark, the contribution proportional to  $\rho_2$  is related by reparameterization invariance [28] to the  $\lambda_2$  contribution. It can be obtained by the replacement

$$\lambda_2 \rightarrow \lambda_2 - \frac{\rho_2}{m_b},$$

in the results of [9].

Thus, the only term we have to add to the existing literature to obtain the complete expression including all  $1/m_b^3$  contributions is the term originating from the Darwin operator whose matrix element is  $\rho_1$ . This contribution is given by

$$\begin{aligned} \frac{d\mathcal{B}_{\rho_1}}{d\hat{s}} &= \mathcal{B}_0 \rho_1 \left[ \left( I^{(C_9^2 + C_{10}^2)} (|C_9^{\text{eff}}|^2 + C_{10}^2) + I^{C_7^2} |C_7^{\text{eff}}|^2 + I^{C_7 C_9} C_7^{\text{eff}} \text{Re}(C_9^{\text{eff}}) \right) \left[ \frac{1}{\hat{u}^3(\hat{s}, \hat{m}_s)} \right]_* \right. \\ &\quad \left. + I_\delta \delta(\hat{s}_u - \hat{s}) \right], \end{aligned} \quad (\text{B1})$$

with limits of integration defined in (17)

$$\hat{s}_l \leq \hat{s} \leq \hat{s}_u, \quad (\text{B2})$$

with

$$\hat{s}_l = 4\hat{m}_l^2, \quad \hat{s}_u = (1 - \hat{m}_s)^2. \quad (\text{B3})$$

The function  $\hat{u}(\hat{s}, \hat{m}_s) = \sqrt{(\hat{s} - (1 - \hat{m}_s)^2)(\hat{s} - (1 + \hat{m}_s)^2)}$  is singular at the upper limit of integration. To regulate this divergence we defined a “star function”

$$[F(\hat{s}, \hat{m}_s)]_* = \lim_{\beta \rightarrow 0} \left\{ F(\hat{s}, \hat{m}_s) \theta(\hat{s}_u - \hat{s} - \beta) - \delta(\hat{s}_u - \hat{s} - \beta) \int_{\hat{s}_l}^{\hat{s}_u - \beta} d\hat{s} F(\hat{s}, \hat{m}_s) \right\}. \quad (\text{B4})$$

This “star function” is analogous to the common plus distribution.

The functions appearing above are

$$\begin{aligned}
I^{(C_9^2+C_{10}^2)} = & -\frac{2}{9} \left[ (1-\hat{s})^2 (11 + 13\hat{s} - 9\hat{s}^2 + 23\hat{s}^3 + 10\hat{s}^4) \right. \\
& - (50 + 37\hat{s} + 48\hat{s}^2 + 38\hat{s}^3 + 70\hat{s}^4 + 45\hat{s}^5) \hat{m}_s^2 \\
& + (85 + 150\hat{s} + 216\hat{s}^2 + 178\hat{s}^3 + 75\hat{s}^4) \hat{m}_s^4 - 2 (30 + 69\hat{s} + 72\hat{s}^2 + 25\hat{s}^3) \hat{m}_s^6 \\
& \left. + (5 + 19\hat{s}) \hat{m}_s^8 + (14 + 15\hat{s}) \hat{m}_s^{10} - 5 \hat{m}_s^{12} \right] \quad (B5)
\end{aligned}$$

$$\begin{aligned}
I^{C_7^2} = & -\frac{8}{9\hat{s}} \left[ (1-\hat{s})^2 (22 - 7\hat{s} + 9\hat{s}^2 + 19\hat{s}^3 + 5\hat{s}^4) \right. \\
& - (78 - 34\hat{s} - 105\hat{s}^2 + 376\hat{s}^3 - 40\hat{s}^4 + 18\hat{s}^5 - 5\hat{s}^6) \hat{m}_s^2 \\
& + (70 + 67\hat{s} - 222\hat{s}^2 - 188\hat{s}^3 - 32\hat{s}^4 - 15\hat{s}^5) \hat{m}_s^4 \\
& + (50 + 60\hat{s} + 258\hat{s}^2 + 136\hat{s}^3) \hat{m}_s^6 - (110 + 157\hat{s} + 111\hat{s}^2 - 50\hat{s}^3) \hat{m}_s^8 \\
& \left. + (38 + 2\hat{s} - 75\hat{s}^2) \hat{m}_s^{10} + 9 (2 + 5\hat{s}) \hat{m}_s^{12} - 10 \hat{m}_s^{14} \right] \quad (B6)
\end{aligned}$$

$$\begin{aligned}
I^{C_7 C_9} = & -\frac{8}{3} \left[ (1-\hat{s})^2 (3 - \hat{s} + 17\hat{s}^2 - 3\hat{s}^3) \right. \\
& - (10 - 13\hat{s} + 56\hat{s}^2 + 58\hat{s}^3 - 10\hat{s}^4 - 5\hat{s}^5) \hat{m}_s^2 \\
& + (5 - 22\hat{s} + 12\hat{s}^2 - 34\hat{s}^3 - 25\hat{s}^4) \hat{m}_s^4 + 2 (10 + 29\hat{s} + 36\hat{s}^2 + 25\hat{s}^3) \hat{m}_s^6 \\
& \left. - (35 + 67\hat{s} + 50\hat{s}^2) \hat{m}_s^8 + (22 + 25\hat{s}) \hat{m}_s^{10} - 5 \hat{m}_s^{12} \right] \quad (B7)
\end{aligned}$$

$$I_\delta = -\frac{16(1-\hat{m}_s^2)^5}{3\sqrt{1-4\hat{m}_s^2}} \left( C_{10}^2 + \left( C_9^{\text{eff}}(\hat{s}) + 2C_7^{\text{eff}} \right)^2 \right) \quad (B8)$$

Notice that these terms correctly reproduce the expression (26) in the limit  $\hat{m}_s \rightarrow 0$ .

### APPENDIX C: THE MOMENTS UP TO $\mathcal{O}(1/M_B^3)$ WITH A CUT ON THE DILEPTON INVARIANT MASS

We write the moments in the form

$$M^{(m,n)} = \frac{\mathcal{B}_0}{\mathcal{B}_\chi} \left( C_{10}^2 M_{10,10}^{(m,n)} + |C_7^{\text{eff}}|^2 M_{7,7}^{(m,n)} + M_{9,9}^{(m,n)} + C_7^{\text{eff}} M_{7,9}^{(m,n)} \right), \quad (C1)$$

where  $\mathcal{B}_\chi$  is given in Eq. (32). The coefficient  $C_9^{\text{eff}}$  depends on the parameters  $x_0$  and  $s_0$  as explained in section II, so we express the moments  $M_{9,9}^{(m,n)}$  and  $M_{7,9}^{(m,n)}$  as integrals which we evaluate numerically,

$$\begin{aligned}
M_{9,9}^{(m,n)} &= \frac{16}{2M_B} \left( -\frac{1}{\pi} \right) \int_{\hat{m}_s}^{\frac{1}{2}(1-\chi)} dx_0 \int_{\hat{m}_s^2}^{x_0^2} d\hat{s}_0 x_0^m \hat{s}_0^n \left[ \sqrt{x_0^2 - \hat{s}_0} \operatorname{Im} \left\{ \left[ 2(1 - 2x_0 + \hat{s}_0) T_1^{(C_9 \pm C_{10})^2} \right. \right. \right. \\
&\quad \left. \left. \left. + \frac{x_0^2 - \hat{s}_0}{3} T_2^{(C_9 \pm C_{10})^2} \right] \right\} |C_9^{\text{eff}}(x_0, \hat{s}_0)|^2 \right] \\
M_{7,9}^{(m,n)} &= \frac{16}{2M_B} \left( -\frac{1}{\pi} \right) \int_{\hat{m}_s}^{\frac{1}{2}(1-\chi)} dx_0 \int_{\hat{m}_s^2}^{x_0^2} d\hat{s}_0 x_0^m \hat{s}_0^n \left[ \sqrt{x_0^2 - \hat{s}_0} \operatorname{Im} \left\{ \left[ 2(1 - 2x_0 + \hat{s}_0) T_1^{C_7(C_9 \pm C_{10})} \right. \right. \right. \\
&\quad \left. \left. \left. + \frac{x_0^2 - \hat{s}_0}{3} T_2^{C_7(C_9 \pm C_{10})} \right] \right\} \operatorname{Re} \left[ C_9^{\text{eff}}(x_0, \hat{s}_0) \right] \right] \tag{C2}
\end{aligned}$$

For the other contributions we find

$$\begin{aligned}
M_{10,10}^{(1,0)} &= \frac{(1-\chi)^4 (7+8\chi)}{30} + \frac{\lambda_1}{m_b^2} \left( \frac{(1-\chi)^3 (1+\chi)}{3} \right) \\
&\quad - \frac{\lambda_2}{m_b^2} \left( \frac{2(1-\chi)^2 \chi (1+6\chi-4\chi^2)}{3} \right) \\
&\quad - \frac{\rho_1}{m_b^3} \left( \frac{67-30\chi+30\chi^3-35\chi^4-32\chi^5}{45} \right) \\
&\quad + \frac{\rho_2}{m_b^3} \left( \frac{1+10\chi-50\chi^3+55\chi^4-16\chi^5}{5} \right) \tag{C3}
\end{aligned}$$

$$\begin{aligned}
M_{7,7}^{(1,0)} &= -\frac{2(41-60\chi+18\chi^2+4\chi^3-3\chi^4+24\log(\chi))}{9} \\
&\quad - \frac{\lambda_1}{m_b^2} \left( \frac{8(8-9\chi+\chi^3+6\log(\chi))}{9} \right) \\
&\quad + \frac{\lambda_2}{m_b^2} \left( \frac{4(7-2\chi-10\chi^3+5\chi^4+12\log(\chi))}{3} \right) \\
&\quad + \frac{\rho_1}{m_b^3} \left( \frac{8(2-27\chi+19\chi^3+6\chi^4+18\log(\chi))}{27} \right) \\
&\quad + \frac{\rho_2}{m_b^3} \left( \frac{8(13-27\chi+23\chi^3-9\chi^4-6\log(\chi))}{9} \right) \tag{C4}
\end{aligned}$$

$$\begin{aligned}
M_{10,10}^{(2,0)} &= \frac{(1-\chi)^5 (4+5\chi)}{45} + \frac{\lambda_1}{m_b^2} \left( \frac{(1-\chi)^4 (43+67\chi+25\chi^2)}{270} \right) \\
&\quad + \frac{\lambda_2}{m_b^2} \left( \frac{(1-\chi)^3 (13+24\chi-222\chi^2+125\chi^3)}{90} \right) \\
&\quad - \frac{\rho_1}{m_b^3} \left( \frac{(1-\chi)^2 (24-27\chi-3\chi^2+191\chi^3+175\chi^4)}{270} \right) \\
&\quad - \frac{\rho_2}{m_b^3} \left( \frac{(1-\chi)^3 (14-63\chi-306\chi^2+175\chi^3)}{90} \right) \tag{C5}
\end{aligned}$$

$$\begin{aligned}
M_{7,7}^{(2,0)} &= -\frac{2(119-210\chi+120\chi^2-20\chi^3-15\chi^4+6\chi^5+60\log(\chi))}{45} \\
&\quad - \frac{\lambda_1}{m_b^2} \left( \frac{127-150\chi-12\chi^2+44\chi^3-3\chi^4-6\chi^5+84\log(\chi)}{27} \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{\lambda_2}{m_b^2} \left( \frac{9 - 2\chi + 4\chi^2 - 36\chi^3 + 35\chi^4 - 10\chi^5 + 12\log(\chi)}{3} \right) \\
& + \frac{\rho_1}{m_b^3} \left( \frac{127 - 210\chi - 24\chi^2 + 188\chi^3 - 39\chi^4 - 42\chi^5 + 60\log(\chi)}{27} \right) \\
& + \frac{\rho_2}{m_b^3} \left( \frac{39 - 66\chi - 8\chi^2 + 84\chi^3 - 63\chi^4 + 14\chi^5 + 12\log(\chi)}{3} \right)
\end{aligned} \tag{C6}$$

$$\begin{aligned}
M_{10,10}^{(0,1)} &= \frac{\lambda_1}{m_b^2} \left( \frac{(1-\chi)^3 (13 + 19\chi + 8\chi^2)}{30} \right) \\
& + \frac{\lambda_2}{m_b^2} \left( \frac{(1-\chi)^3 (3 + 13\chi - 8\chi^2)}{6} \right) \\
& + \frac{\rho_1}{m_b^3} \left( \frac{(1-\chi)^2 (177 + 254\chi + 201\chi^2 + 88\chi^3)}{90} \right) \\
& + \frac{\rho_2}{m_b^3} \left( \frac{(1-\chi)^2 (1 - 18\chi - 47\chi^2 + 24\chi^3)}{10} \right)
\end{aligned} \tag{C7}$$

$$\begin{aligned}
M_{7,7}^{(0,1)} &= -\frac{\lambda_1}{m_b^2} \left( \frac{2(23 - 12\chi - 18\chi^2 + 4\chi^3 + 3\chi^4 + 24\log(\chi))}{9} \right) \\
& - \frac{\lambda_2}{m_b^2} \left( \frac{2(31 - 28\chi - 18\chi^2 + 20\chi^3 - 5\chi^4 + 24\log(\chi))}{3} \right) \\
& + \frac{\rho_1}{m_b^3} \left( \frac{2(77 - 132\chi - 54\chi^2 + 76\chi^3 + 33\chi^4 - 120\log(\chi))}{27} \right) \\
& + \frac{\rho_2}{m_b^3} \left( \frac{2(281 - 324\chi - 54\chi^2 + 124\chi^3 - 27\chi^4 + 168\log(\chi))}{9} \right)
\end{aligned} \tag{C8}$$

$$\begin{aligned}
M_{10,10}^{(0,2)} &= -\frac{\lambda_1}{m_b^2} \left( \frac{4(1-\chi)^5 (4 + 5\chi)}{135} \right) \\
& - \frac{\rho_1}{m_b^3} \left( \frac{2(1-\chi)^4 (31 + 64\chi + 40\chi^2)}{135} \right) \\
& + \frac{\rho_2}{m_b^3} \left( \frac{2(1-\chi)^4 (1 - 26\chi + 10\chi^2)}{45} \right)
\end{aligned} \tag{C9}$$

$$\begin{aligned}
M_{7,7}^{(0,2)} &= \frac{\lambda_1}{m_b^2} \left( \frac{8(119 - 210\chi + 120\chi^2 - 20\chi^3 - 15\chi^4 + 6\chi^5 + 60\log(\chi))}{135} \right) \\
& + \frac{\rho_1}{m_b^3} \left( \frac{8(139 - 60\chi - 210\chi^2 + 140\chi^3 + 15\chi^4 - 24\chi^5 + 120\log(\chi))}{135} \right) \\
& - \frac{\rho_2}{m_b^3} \left( \frac{16(73 - 165\chi + 165\chi^2 - 100\chi^3 + 30\chi^4 - 3\chi^5 + 30\log(\chi))}{45} \right)
\end{aligned} \tag{C10}$$

$$\begin{aligned}
M_{10,10}^{(1,1)} &= \frac{\lambda_1}{m_b^2} \left( \frac{(1-\chi)^4 (23 + 62\chi + 50\chi^2)}{270} \right) \\
& + \frac{\lambda_2}{m_b^2} \left( \frac{(1-\chi)^4 (13 + 82\chi - 50\chi^2)}{90} \right)
\end{aligned}$$

$$\begin{aligned}
& + \frac{\rho_1}{m_b^3} \left( \frac{(1-\chi)^3 (71 + 183\chi + 276\chi^2 + 190\chi^3)}{270} \right) \\
& - \frac{\rho_2}{m_b^3} \left( \frac{(1-\chi)^3 (13 + 9\chi - 252\chi^2 + 110\chi^3)}{90} \right)
\end{aligned} \tag{C11}$$

$$\begin{aligned}
M_{7,7}^{(1,1)} = & - \frac{\lambda_1}{m_b^2} \left( \frac{4 (2 + 15\chi - 33\chi^2 + 16\chi^3 + 3\chi^4 - 3\chi^5 + 6 \log(\chi))}{27} \right) \\
& - \frac{\lambda_2}{m_b^2} \left( \frac{4 (10 - 13\chi - \chi^2 + 8\chi^3 - 5\chi^4 + \chi^5 + 6 \log(\chi))}{3} \right) \\
& + \frac{\rho_1}{m_b^3} \left( \frac{4 (62 - 75\chi - 195\chi^2 + 280\chi^3 - 15\chi^4 - 57\chi^5 - 30 \log(\chi))}{135} \right) \\
& + \frac{\rho_2}{m_b^3} \left( \frac{4 (257 - 285\chi - 195\chi^2 + 400\chi^3 - 210\chi^4 + 33\chi^5 + 150 \log(\chi))}{45} \right)
\end{aligned} \tag{C12}$$

## REFERENCES

- [1] B.Grinstein, M.Savage, and M.B.Wise, Nucl. Phys. **B319** (1989) 271.
- [2] M. Misiak, Nucl. Phys. **B393** (1993) 23; **Erratum**, Nucl. Phys. **B439** (1995) 461.
- [3] A.J. Buras and M.Münz, Phys. Rev. D **52** (1995) 186.
- [4] A.Ali, G.F.Giudice, and T.Mannel, Z.Phys.**C67** (1995) 417; C.Huang, W.Liao, and Q.Yan, Phys. Rev. D **59** (1999) 011701; P.Cho, M.Misiak, and D.Wyler, Phys. Rev. D **54** (1996) 3329; J.Hewett and J.D.Wells, Phys. Rev. D **55** (1997) 5549; T.Goto, Y.Okada, Y.Shimizu, and M.Tanaka, Phys. Rev. D **55** (1997) 4273; E.Lunghi, A.Masiero, I.Scimemi, and L.Silvestrini, hep-ph/9906286.
- [5] R. Ammar *et al.* (CLEO Collaboration), Phys. Rev. Lett. **71** (1993) 674.
- [6] M.S. Alam *et al.* (CLEO Collaboration), Phys. Rev. Lett. **74** (1995) 2885; hep-ph/9908022.
- [7] J.Hewett, hep-ph/9406302; T.G.Rizzo, Phys. Rev. D **58** (1998) 114014; F.Borzumati and C.Greub, Phys. Rev. D **58** (1998) 074004; W.de Boer *et al.*, Phys. Lett. B **438** (1998) 281; H.Baer *et al.*, Phys. Rev. D **58** (1998) 015007; M.Ciuchini *et al.*, Nucl. Phys. **B527** (1998) 21; J.Agrawal *et al.*, *Int. Jour. Mod. Phys. A* **11** (( ) 1996) 2263; M.A.Diaz *et al.*, Nucl. Phys. **B551** (1999) 78; G.V.Kranrotis, Z.Phys.**C71** (1996) 163; A.L.Kagan and M.Neubert, *Eur.Phys.J.***C7** (1999) 5.
- [8] S. Glenn *et al.* (CLEO Collaboration), Phys. Rev. Lett. **80** (1998) 2289; B.Abbott *et al.* (DØ Collaboration), Phys. Lett. B **423** (1998) 419; C.Albajar *et al.* (UA1 Collaboration), Phys. Lett. B **262** (1991) 163. Also, a recent search in exclusive channels: T.Affolder *et al.* (CDF Collaboration), hep-ex/9905004.
- [9] A. Ali, L.T.Handoko, G.Hiller, T.Morozumi, Phys. Rev. D **55** (1997) 4105.
- [10] A. Ali and G. Hiller, Phys. Rev. D **58** (1998) 074001.
- [11] B.Grinstein, M.Savage, and M.B.Wise, Nucl. Phys. **B319** (1989) 271.
- [12] Z.Ligeti, Y.Nir, Phys. Rev. D **49** (1994) 4331; M.Gremm, A.Kapustin, Z.Ligeti, M.Wise, Phys. Rev. Lett. **77** (1996) 20; M.Neubert, Phys. Lett. B **389** (1996) 727.
- [13] C. Bauer and C.Burrell, Phys. Lett. B **469** (1999) 248.
- [14] J. Chay, H. Georgi and B. Grinstein, Phys. Lett. **B247** (1990) 399.
- [15] I.I. Bigi, M. Shifman, N.G. Uraltsev, A. Vainshtein, Phys. Rev. Lett. **71** (1993) 496, B. Blok, L. Koyrakh, M. Shifman, A. Vainshtein, Phys. Rev. **D49** (1994) 3356, Erratum-ibid. **D50** (1994) 3572, A. V. Manohar, M. B. Wise, Phys. Rev. **D49** (1994) 1310; I.I.Bigi, N.G.Uraltsev, A.I.Vainshtein, Phys. Lett. B **293** (1992) 430; **Erratum**, Phys. Lett. B **297** (1993) 477.
- [16] A.Falk, M.Luke, and M.Savage, Phys. Rev. D **49** (1994) 3367.
- [17] C. Bauer, Phys. Rev. **D57** (1998) 5611; **Erratum** Phys. Rev. D **60** (1999) 099907.
- [18] M. Gremm and A. Kapustin, Phys. Rev. **D55** (1997) 6924.
- [19] A.F. Falk and M.Neubert, Phys. Rev. **D47** (1993) 2965; Phys. Rev. D **47** (1993) 2982.
- [20] T. Mannel, Phys. Rev. **D50** (1994) 428.
- [21] A. F. Falk, M. Luke, M. Savage, Phys. Rev. **D53** (1996) 2491.
- [22] M.A. Shifman and M.B. Voloshin, Sov. J. Nucl. Phys. 45 (1987) 292; M.B. Voloshin, N.G. Uraltsev, V.A. Khoze and M.A. Shifman, Sov. J. Nucl. Phys. 46 (1987) 112.
- [23] B. Blok, R.D. Dikeman and M. Shifman, Phys. Rev. **D51**, 6167 (1995).
- [24] C. Bauer, A. Falk and M. Luke, Phys. Rev. D **54** (1996) 2097.
- [25] G.Buchalla and G.Isidori, Nucl. Phys. **B525** (1998) 333.

- [26] M. Neubert, Phys. Rev. **D49** (1994) 3367; Phys. Rev. **D49** (1994) 4623.
- [27] A. Falk and M. Luke, Phys. Rev. D **57** (1998) 424.
- [28] M.Luke and A.Manohar, Phys. Lett. B **286** (1992) 348.